Interconnection Negotiations between Telecommunication Networks and Universal Service Objectives

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Abstract

We study negotiations between Telecommunication Networks over access fees, that is, fees one network has to pay to another one, whenever a customer of the former places a phone call to a customer of the latter. We show that revenue generated from agreed access fees can alleviate the problem of providing telecommunication services at reasonable prices at rural and other high cost areas. The model consists of three interconnected networks - two located in the low cost area (urban area) and one located in the high cost (rural area) - who negotiate pair-wise over access fees. If urban customers place high value to being able to reach rural customers, then the rural network's revenue from selling access will be high enough so that it will become profitable given interconnection, even though it would be making losses if it were not interconnected with the urban networks. The reason is that access fees allow market participants to internalize some of the network externalities. The results are robust with respect to the timing of the moves of the game and, more importantly, with respect to the degree of competition in the urban market. The lessons apply to other industries that have the network structure, for instance electricity, postal services and transportation.

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1 Introduction

This paper addresses the issue of providing universal service in a deregulated network. Many industries that have a network structure, for instance gas, electricity, telecommunications and postal services, exhibit large cost discrepancies. The provision of the 'same' service may cost as much as ten times more in some parts of the network than in others. When this is the case, it is profitable to operate only in some areas. In most countries, till relatively recently, these industries were either state-owned monopolies or regulated by the government. Under these ownership structures the government was able to cross-subsidize high-cost with low-cost, profitable areas.[1] Currently the markets for telecommunications services, gas and electricity in the United States, and in other countries are undergoing major changes. Most of the firms operating in these industries have been privatized and regulation is being replaced by competition. As many have observed, see for instance Baumol (1999), direct cross-subsidies may be incompatible with competition and deregulation.

The objective of this paper is to demonstrate how indirect cross-subsidization can take place in a deregulated environment. More specifically, we would like to identify market conditions that ensure universal coverage of telecommunication services without government subsidies. Most of the analysis will be based on the telecommunication industry, but a lot of the insights obtained carry over to other industries as well.

After the introduction of the Telecommunications Act of 1996 by the Congress, the market for local and long distance service is opened to competition. The major technological breakthroughs in the telecommunication and other related industries changed the structure of the telecommunication markets. Due to these technological advances the distinction between telecommunication and information services, such as access to the information superhighways, became unclear. This created the need to change the institutions that govern telecommunication markets in order to avoid creating distortions. According to the previous arrangement the provision of local telephone services was delivered by local monopolies, the Regional Bell Operating Companies, (RBOCs), which were subject to regulation. On the other hand, the markets for long distance services and for internet access were open to competition. The Congress in order to accommodate the technological advancements passed the Telecommunications Act of 1996 which opened all telecommunication markets to competition. This Act contains a clause that states that advanced telecommunication and information services must be available to all Americans at reasonable rates. This clause is included into what the Federal Communication Commission, FCC, refers to as 'universal service objectives,' USOs.

The fact that there exist big cost discrepancies in providing telecommunication services in different areas, has lead many to believe that deregulation will threaten universal service objectives. One would

[1] See Maria Maher (1999) for an empirical study which shows that there are economies of scale in the provision of access to the local telecommunication network and that costs differ by geographical location.

expect that no firm will be willing to provide services in high cost areas, such as remote and sparsely populated rural areas, whereas competition will prevail in densely populated and more profitable urban areas. Under the previous arrangement the prices charged by the regulated RBOC reflected a cross subsidy from urban to rural markets. This kind of direct cross subsidization is likely to be impossible in a deregulated environment. The Congress realizing this problem asked the FCC to form a Federal-State Joint Board to specify the services that should be included in the universal service objectives and ways to financially support them, [16]. Similar documents have been prepared by Oftel, [25], in the UK and by the European Commission, [15], among many others. The major priority of the universal service objectives is the provision of advanced telecommunication and information services to rural customers at prices that are comparable to the ones that urban customers pay.

The goal of this research is to show that this objective can be achieved without outside subsidies in a deregulated[2] market of telecommunication services. The reason is positive network externalities. The value generated from serving a rural customer does not only include his willingness to pay for telephone service, but also the utility that other customers enjoy from being able to reach him. If urban customers, residential and businesses, like to place many phone calls in the rural market, then the urban networks may find it profitable to subsidize the rural ones. Providing service to rural customers increases the profits of a network serving an urban area, since urban customers can place more phone calls. Interconnection enlarges the market and the benefit of having more customers connected accrues to all parts of the network. Similar observations hold for other industries, for instance postal services. Delivering a letter to a relative in a remote area generates surplus to the receiver as well as to the sender.

Our model consists of two different markets; a rural or high cost area and an urban or low cost area. There is only one network operating in the rural market. In the urban market we consider three alternative scenarios. First we look at the case there is only one urban network, (monopoly), and then we consider the case of a competitive urban market, and finally the more realistic case, where there are two horizontally differentiated urban networks. We assume that networks choose their prices non-cooperatively and they bargain pairwise over the level of access fees. The outcome of negotiations is determined by the Nash-Bargaining solution. We demonstrate that interconnection is a feature of the market that may support universal service, even in a deregulated market.[3] How? The rural network generates revenue by selling services to rural customers AND by selling access to its facilities to the urban networks. Urban networks pay an access fee to the rural network each time an urban customer places a call to a rural customer (and the opposite). Under certain conditions net access revenue of a rural network can lead to positive profits, even if this network would make losses if it were isolated (that is, in the case that there were no calls made

[2] For an excellent survey of the enormous literature on regulation see Armstrong and Sappington (2004).

[3] 'Interconnection' means that customers of one network can reach customers of the other network.

between urban and rural customers).

When we examine the case of horizontally differentiated urban networks, our model builds on Armstrong (1998), Carter and Wright (1999), and Laffont, Rey and Tirole (1998a). These papers study, among others, competition between telecommunication networks in a deregulated or partially regulated environment. Their focus is on the determination of the price of telecommunications services and of the access fees that each network pays to competing networks, when a customer of the former places a call to a customer of the latter. We extend the basic model by studying the determination of access fees and prices in an environment that consists of three interconnected networks. There is two-way access between all networks and competition for customers between the networks located in the urban area. This model has the advantage that it builds upon the standard network-interconnection model - which makes our analysis directly comparable to the existing ones; the downside is that it does not allow for tractable analytical results. We calculate the equilibrium prices and access fees in a parametrized example and show that in many instances, even though the rural network would be making losses if it were not interconnected with the urban networks, it will become profitable if there is interconnection. The urban network subsidizes the rural network in the following way: whenever a rural customer places a call to an urban customer the urban network charges a *negative* access fee to the rural network. This result is robust to different sequence of moves of the game.

Our results demonstrate that if urban customers place a large enough fraction of their total demand for calls to rural customers, then the rural network generates revenue from selling access that is high enough to cover its losses from providing telecommunication services. If this is the case in some markets, then there is no need to arrange for outside subsidies, since the market is viable on its own. One would think that such an insight that is based on 'voluntary' cross subsidization, between the urban and the rural market, would be vulnerable to the degree of competition in the urban market, the argument being that competition erodes profits, and there is nothing left for 'cross-subsidization.' Our results suggest that this is not true at least in the case where there is only one network serving the rural area, (which given large fixed costs is probably the scenario that is most likely). Actually it is exactly the opposite. The monopoly power of the rural network vis-a-vis urban networks that offer indistinguishable services, and hence have no market power, makes it a very tough negotiator. When the rural network bargains with an urban network over access fees, the rural network knows that if they fail to reach agreement, the urban network will suffer a severe loss in market share, (in the extreme case we solved analytically it will be actually kicked out of the market). The reason being that no urban customer would like to pay the same price for telephone service and not being able to complete phone calls across markets, when he/she can buy service from some other network that has interconnection with the rural market. On the other hand, the rural network does not loose anything in the event that it fails to reach agreement with a particular urban network, since it can still connect with the other urban networks who now serve all the market. This mechanism, that is the effect of interconnection agreements on market shares, is also present in the case that urban networks have some market power, but is less extreme. In those cases though there is different channel for potential allowing sufficient subsidizations: the higher the market power the higher the profits of urban networks. But higher market power also means higher bargaining power vis-a-vis the rural network, that is higher market power in the urban market implies higher profits but also stronger bargaining position, whereas low market power implies very low profits in the urban market but at the same time, very weak position in the negotiations. In other words depending on the market power in the urban market there are different mechanisms in place that work in favor of cross-subsidization. A weakness of the current analysis is that our results are completely silent to whether prices will be comparable across markets. In the future we plan to investigate the effect of cross-market price constraints, similar to the ones in [1], on the negotiated access fees and on the equilibrium profits.

There has been some recent theoretical work that studies universal service objectives issues but with different focus then the current project. Anton, Weide and Vettas (2002) develop a multi-market model that consists of an oligopolistic urban market, entry auctions for rural service and cross market price restrictions. They analyze how these restrictions affect pricing in the urban market and entry decisions in the rural market. The assumed market structure does not consist of networks so there is no other interconnection between the rural and the urban market. Laffont and Tirole (2000) in Chapter 6 discuss Universal Service Obligations. They offer some institutional background and analyze ways to generate the "right subsidies" by means of designing auction mechanisms that determine, among, others the structure of the market, (auction mechanisms with endogenous market structure). Milgrom (1996) is the first to discuss the application of mechanisms with endogenous market structure, developed by Dana and Spier (1994), in Universal Service Obligations.[4] Compared to the previous work, our focus is not how to generate the right subsidies for USOs, but to demonstrate that outside intervention and subsidies may not be necessary. Baumol (1999) and Armstrong (2001a), like this paper, are concerned with the issue of how competition will affect USO's. Both consider the stage of transition to competition, where there is an incumbent network who faces potential entrants that may or may not bypass its facilities. The incumbent does not need to buy access to the entrant's facilities. The focus of those papers is to determine the access fees paid to the incumbent by the entrant that generate efficient entry, and when bypass is possible, efficient make-or-buy decisions for the entrant. Here we examine the case of mature competition where all networks need to purchase necessary inputs from each other. The papers by Riordan (2000) and (2001) investigate the economic rational for USO's. In this paper we do not aim at exploring whether USO's is indeed an

[4] See also Weller (1998).

appropriate objective, but we show that it can be achieved even in a deregulated environment.

2 Unit Demands

2.1 Monopoly in the Urban Market

2.1.1 The model

We will start by examining a very simple scenario. We consider a situation where there are two telecommunication networks, A and R. Network A operates in the low cost area (or urban area) . Network R is located in the high cost area, the rural area. When a subscriber of A calls a subscriber of R, A has to 'buy' access to R's subscribers. In short, network i sells access to network j and the opposite. Hence there is 'two-way' access without competition between network R and network $i = A, B$. In order for urban customers to be able to terminate a call in the rural market, and the reverse, the urban and the rural network must sign an interconnection agreement.

- **Demand Structure in the Urban Market**

We assume that

- consumers derive utility only from placing and not from receiving phone calls.

The utility of an urban consumer connected to network A is given by

$$U(p_A) = v - p_A.$$

In the **rural** market there is a single network called R. The net surplus for a rural customer from being connected to the network is given by

$$U(p_R) = v - p_R.$$

We consider the case where demand for phone calls in both markets is fixed and equal to one. A rural customer places a fraction r_R of the total calls within the rural market and a fraction $(1-r_R)$ to the urban market. Similarly, an urban customer places a fraction r_U of the total calls within the urban market and a fraction $(1-r_U)$ to the rural market. A result of this assumption is that the demand of network A depends on whether it has signed an interconnection agreement with R.

In the case that A signs interconnection agreements with R, its demand is given by

$$d_A(p_A) = \begin{cases} 1 \text{ if } v - p_A \geq 0 \\ 0 \text{ otherwise} \end{cases},$$

where p_A stands for the price charged by A in case it has signed an interconnection agreement with R. In case of disagreement with R demand is given by

$$d_A^D(p_A^D) = \begin{cases} r_u \text{ if } v - p_A^D \geq 0 \\ 0 \text{ otherwise} \end{cases},$$

where p_A^D denotes the price charged by A in case it fails to sign an interconnection agreement with R. Similarly, in case of agreement with A, the demand for the rural network is given by

$$d_R(p_R) = \begin{cases} \gamma \text{ if } v - p_R \geq 0 \\ 0 \text{ otherwise} \end{cases},$$

where $\gamma \in [0,1]$ and p_R stands for the price that the rural network charges in case of agreement with A. Let p_R^D denote the price charged by the rural network in case of disagreement with A. In this event, the demand of the rural network is given by

$$d_R^D(p_R^D) = \begin{cases} \gamma r_R \text{ if } v - p_R^D \geq 0 \\ 0 \text{ otherwise} \end{cases}.$$

- **Cost Structure and Profits**

Now we specify the cost structure of the networks. Let[5] c_i^o denote the cost of originating a phone call from network i, and c_i^T the cost of terminating a phone call at network i, $i = A, R$. Network i incurs a fixed cost c_i^F of servicing a customer.

We use p_i to denote the price set by network i and t_{AR} , (t_{RA}), to denote the access fee that the rural network $R, (A)$, pays to network A, (R), each time a subscriber of network R, (A), calls a subscriber of network A, (R).

Remark: We will argue in a minute that networks will not charge different prices for phone calls completed within the network and phone calls completed across networks.

Given this observation network $A's$ and $R's$ profits are given by

$$\begin{aligned} \Pi_A &= p_A - c_A^0 - r_U c_A^T - (1 - r_R)c_A^T - (1 - r_U)t_{RA} + (1 - r_R)\gamma t_{AR} - c_A^F \\ \Pi_R &= \gamma(p_R - c_R^0) - r_R \gamma c_R^T - (1 - r_U)c_R^T - (1 - r_R)\gamma t_{AR} + (1 - r_U)t_{RA} - c_R^F \end{aligned}$$

Let

$$\begin{aligned} \pi_A(p) &= \left(p - c_A^o - c_A^T\right), \text{ and} \\ \pi_R(p) &= \left(p - c_R^o - c_R^T\right), \end{aligned}$$

[5] The cost structure is similar to the one in Armstrong (1998).

then $A's$ profits can be rewritten as

$$\Pi_A = \underbrace{\pi_A(p_A)}_{\text{profits from retail}} + \underbrace{\left[(1-r_R)\gamma\left(t_{AR}-c_A^T\right)-(1-r_u)\left(t_{RA}-c_A^T\right)\right]}_{\text{net access revenue in the urban market}} - \underbrace{c_A^F}_{\text{fixed costs}}$$

The first term in the above expression is firm $A's$ profit in the retail sector and the second term is the firm's profit from buying or selling access in the rural market. The profits of the network serving the rural area are given by

$$\Pi_R = \underbrace{\pi_R(p_R)}_{\text{profits from retail}} + \underbrace{\left[(1-r_u)\left(t_{RA}-c_R^T\right)-(1-r_R)\gamma\left(t_{AR}-c_R^T\right)\right]}_{\text{net access revenue in the rural market}} - \underbrace{c_R^F}_{\text{fixed costs}}.$$

Assumption 1: We will assume that $v > c_R^T$ and that $v > c_A^T$, that is the value of a call to consumers is greater than the cost of terminating a call of both networks. Moreover we will assume that the rural network's termination cost is as least as large as the one of the urban network, that is $c_R^T \geq c_A^T$.

Assumption 2: We set $c_A^o = c_R^o = c_A^F = 0$. This normalization is done for expositional simplicity.

2.1.2 Determination of Prices and Access fees

We study the equilibrium choices of prices and access fees, in a sequential move game where networks first negotiate over access fees and then set their prices non-cooperatively.

Stage 1: Networks A and R negotiate over access fees (t_{RA}, t_{AR}).

Stage 2: Network j sets its price p_j, $j = A, R$.

We are looking for a subgame perfect equilibrium, so we will start solving the game backwards.

Stage 2: Setting Prices

Since A and R are local monopolists and consumers have inelastic demand, they will extract all the surplus from consumers. The same would be true even if networks were allowed to price discriminate based on where the phone call would be terminated.

Lemma 1 *The prices set by A and R in case of agreement and in case of disagreement are given by*

$$\begin{aligned} p_A &= v = p_A^D \\ p_R &= v = p_R^D. \end{aligned}$$

Proof: A and R face no competition. In this unit demand model the networks in both markets set a price that extracts all consumer surplus, that is

$$\begin{aligned} p_A &= v = p_A^D \\ p_R &= v = p_R^D, \end{aligned}$$

where p_j, $j = A, R$ is network $j's$ price in case of agreement and p_j^D is $j's$ price in case of disagreement.

Stage 1: Negotiations

We examine the determination of access fees through free negotiations without regulation. Networks will be assumed to negotiate pairwise over access fees. The outcome of negotiations is determined by the Nash Bargaining Solution. The US Telecommunication Act of 1996 states that access fees should be negotiated among networks subject to regulatory approval. In Europe interconnection agreements should be negotiated within the framework of the European Law and the supervision of the National Regulatory Agencies. In other countries, for instance in New Zealand, after the privatization of the national telecommunication provider, there is no regulatory agency and networks should reach interconnection agreements that do not violate the existing antitrust laws.

The Nash Bargaining solution is a cooperative solution concept, that allows for a parsimonious representation of the conflicts of interests in a given negotiation. According to the Nash Bargaining Solution, negotiating parties split the 'gains from reaching an agreement', equally among each other. For a discussion of the application of the Nash Bargaining Solution to various bargaining problems and its relationship with non-cooperative dynamic solution concepts see Binmore, Rubinstein and Wolinsky (1986).

The access fees should solve:

$$(t_{AR}, t_{RA}) \in \arg\max(\Pi_A - \Pi_A^D)(\Pi_R - \Pi_R^D). \tag{1}$$

The payoffs that accrue to A and R respectively in case of agreement are given by

$$\Pi_A = (v - c_A^T) - R_{AR} - [\gamma(1 - r_R) - (1 - r_U)]c_A^T$$

and

$$\Pi_R = \gamma(v - c_R^T) - c_R^F + R_{AR} + [\gamma(1 - r_R) - (1 - r_U)]c_R^T$$

where

$$R_{AR} = (1 - r_U)t_{RA} - (1 - r_R)t_{AR}\gamma.$$

In case the networks fail to reach an interconnection agreement their payoffs are given by where

$$
\begin{aligned}
\Pi_A^D &= r_U(v - c_A^T) \\
\Pi_R^D &= \gamma r_R(v - c_R^T) - c_R^F.
\end{aligned} \tag{2}
$$

The first order conditions of (1) reduce to

$$\Pi_A - \Pi_A^D = \Pi_R - \Pi_R^D,$$

from which we can obtain the following:

Proposition 1 *The rural network's net access revenue from interconnection is given by*

$$R_{AR} = 0.5\left[\left((1 - r_U) - (1 - r_R)\gamma\right)v - c_A^T\gamma(1 - r_R) + c_R^T(1 - r_U)\right]$$

Proof. See Appendix. ■

Proposition 2 *If* $(1 - r_U) > (1 - r_R)\gamma$ *then* $R_{AR} > 0$.

Proof. Recall that

$$
\begin{aligned}
R_{AR} &= 0.5\left[\left((1 - r_U) - (1 - r_R)\gamma\right)v - c_A^T\gamma(1 - r_R) + c_R^T(1 - r_U)\right] \\
&\geq 0.5\left[\left((1 - r_U) - (1 - r_R)\gamma\right)v - c_R^T\gamma(1 - r_R) + c_R^T(1 - r_U)\right] \\
&= 0.5\left[\left((1 - r_U) - (1 - r_R)\gamma\right)v + c_R^T((1 - r_U) - \gamma(1 - r_R))\right] \\
&= 0.5\left((1 - r_U) - (1 - r_R)\gamma\right)(v + c_R^T),
\end{aligned}
$$

where the last line is non-negative so long as $(1 - r_U) > (1 - r_R)\gamma$. ■

Proposition 3 *The equilibrium profits of the networks are given by*

$$
\begin{aligned}
\Pi_R = {} & 0.5\gamma(1 + r_R)(v - c_R^T) + 0.5(1 - r_U)(v - c_A^T) - c_R^F \\
& -0.5\left[(1 - r_U) - \gamma(1 - r_R)\right](c_R^T - c_A^T)
\end{aligned} \tag{3}
$$

and

$$
\begin{aligned}
\Pi_A = {} & 0.5(1 + r_U)(v - c_A^T) + 0.5\gamma(1 - r_R)(v - c_R^T) \\
& -0.5\left[(1 - r_U) - \gamma(1 - r_R)\right](c_R^T - c_A^T)
\end{aligned} \tag{4}
$$

Proof: See Appendix

Remark 1 *The rural network's equilibrium profits will be non-negative provided that*

$$
\begin{aligned}
& 0.5\left(\gamma(1+r_R)+(1-r_U)\right)v \\
\geq \;& c_R^F + c_R^T\left(\gamma r_R + 0.5(1-r_U)\right) + 0.5\gamma(1-r_R)c_A^T
\end{aligned}
$$

Remark 2 *Both networks benefit from interconnection. From (2) and (3), (4) we obtain that*

$$\Pi_R - \Pi_R^D = 0.5\gamma(1-r_R)(v-c_A^T) + 0.5(1-r_U)(v-c_R^T) \geq 0. \tag{5}$$

For network A one can obtain

$$\Pi_A - \Pi_A^D = 0.5(1-r_U)(v-c_A^T) + 0.5(1-r_U)(v-c_R^T) \geq 0. \tag{6}$$

From (5) and (6 we see that both networks benefit from interconnection.

We proceed to examine the effect of parameters on the equilibrium profits of the networks.

Proposition 4 *The equilibrium profits of the rural network are (i) decreasing in r_U and (ii) increasing in r_R if $0.5v+0.5c_A^T \geq c_R^T$. The urban network´s equilibrium profits are (i) increasing in r_U and (ii) decreasing in r_R.*

Proof. Partially differentiating with respect to the corresponding parameters we obtain

$$\frac{\partial \Pi_R}{\partial r_U} = -v0.5 - 0.5c_R^T < 0 \,\text{since}\; v > c_R^T$$

$$
\begin{aligned}
\frac{\partial \Pi_R}{\partial r_R} &= 0.5v\gamma - c_R^T\gamma + 0.5c_A^T\gamma \geq 0 \\
\text{if } 0.5v + 0.5c_A^T &\geq c_R^T.
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial \Pi_A}{\partial r_U} &= 0.5v + 0.5c_R^T - c_A^T > 0 \text{ since } v > c_R^T \geq c_A^T \\
\frac{\partial \Pi_A}{\partial r_R} &= -0.5\gamma v + 0.5\gamma c_A^T < 0, \text{ since } v > c_A^T.
\end{aligned}
$$

■

The profits of the rural network are decreasing in r_U, where r_U stands for the fraction of urban phone calls completed within the urban market. Also Π_R are increasing in r_R so long as the termination costs in the rural network, c_R^T are not too high. Finally as expected, Π_R is increasing in v and decreasing in c_R^T and c_R^F, that is

$$\frac{\partial \Pi_R}{\partial v} > 0, \; \frac{\partial \Pi_R}{\partial c_R^T} < 0, \text{ and } \frac{\partial \Pi_R}{\partial c_R^F} < 0.$$

Contrary to the rural network, the urban network's profits are increasing in r_U and decreasing in r_R. We will later establish that the qualitative results obtained in this very simple model match the ones we will obtain when we study via simulations a more elaborate model that was based on the standard 2 - way network interconnection model.

In this section we looked at a polar case where the urban network is a monopolist in the market. In order to see whether the obtained results are robust to other market structures, we examine first the case of a competitive urban market and then the case where networks have some market power.

2.2 Competitive Urban Market

2.2.1 The Model

We examine the scenario where two networks operating in the urban market, call them A and B offer indistinguishable services. Consumers have inelastic demand for phone calls. We normalize the number of phone calls demanded by consumers in the urban market to one. A fraction r_U of these calls are completed within the urban market and a fraction $(1 - r_U)$ are directed to the rural market.

Assumption: **Balanced calling pattern:** A customer of A calls with equal probability a subscriber of A and a subscriber of B.

Let s denote $A's$ market share in the urban market. Networks A and B charge different prices for calls terminating in different networks. We use p_i to denote the price charged by i for calls completed within i; p_i^j stands for the price charged by i for the calls completed in network j.

Consumer Preferences and Demand

In the **urban** market consumers may subscribe to just one network. We assume that r_U is the fraction of total phone calls completed within the urban market and $(1 - r_U)$ is the percentage of total phone calls that are directed to the rural market. Consider a consumer subscribing to i, that her indirect utility

$$\psi_i(p_i, p_i^j, p_i^R, m) = sr_U(v - p_i) + (1 - s)r_U(v - p_i^j) + s(1 - r_U)(v - p_i^R) + m, \; i, j \in \{A, B\}$$

where m represents the consumption of other goods. Note that in this model 1 denotes the aggregate quantity of phone calls that the average subscriber of network i plans to make. Some of these phone calls are completed within the urban market , whereas some are made to customers in the rural market.

In the **rural** market there is a single network called R and a measure $\gamma \in [0,1]$ of identical consumers.. Again demand for phone calls of a representative consumer is fixed and equal to one. We assume that a rural customer places a fraction r_R of the total calls within the rural market and a fraction $(1-r_R)$ to the urban market. The net surplus for a rural customer from being connected to the network is given by

$$\psi_R(p_R, m) = \gamma r_R (v - p_R) + \gamma(1-r_R)s(v - p_R^A) + \gamma(1-r_R)(1-s)(v - p_R^A).$$

Note that the rural network faces no competition, hence will charge a price equal to v no matter where the phone call is terminated, that is there will be no price discrimination based on where a phone call is terminated. A consumer places a phone call only if $v \geq p$, where p is the price of some phone call.

Demand for phone calls hence depends as usual on price and also on the interconnection agreements. For instance in the case that A signs interconnection agreements with all networks its demand is given by

$$d_A(p_A) = 1 \text{ if } v - p_A \geq 0; v - p_A^R \geq 0 \text{ and } v - p_A^B \geq 0\ ,$$

In case of disagreement with R demand is given by

$$d_A^{DAR}(p_A^{DAR}) = r_u \text{ if } v - p_A^D \geq 0; v - p_A^R \geq 0 \text{ and } v - p_A^B \geq 0,$$

and so forth. In an analogous way we can find the demand for all possible ranges of prices and all possible interconnection scenaria, for all urban networks as well as for network R. For instance in case of agreement with A, and B the demand for the rural network is given by

$$d_R(p_R) = \gamma \text{ if } v - p_R \geq 0; v - p_R^A \geq 0 \text{ and } v - p_R^B \geq 0,$$

Cost Structure and Profits

The cost structure is the same as in the previous scenario, that is a network i, $i = A, B, R$, incurs a cost c_i^o of originating a phone call, a cost c_i^T of terminating a phone call and a fixed cost c_i^F. The profits of A and B are given by

$$\begin{aligned}\Pi_A &= s^2 r_u(p_A - c_A^0 - c_A^T) + s(1-s)r_u(p_A^B - c_A^O - t_B) \\ &\quad + s(1-r_u)(p_A^R - c_A^T - t_{RA}) - c_A^F + \\ &\quad s(1-s)r_u(t_A - c_A^T) + s(1-r_R)\gamma(t_{AR} - c_A^T)\end{aligned}$$

$$\begin{aligned}\Pi_B &= (1-s)^2 r_u(p_B - c_B^0 - c_B^T) + s(1-s)r_u(p_B^A - c_B^O - t_A) \\ &\quad + (1-s)(1-r_u)(p_B^R - c_B^T - t_{RB}) - c_B^F + \\ &\quad s(1-s)r_u(t_B - c_B^T) + (1-s)(1-r_R)\gamma(t_{BR} - c_B^T)\end{aligned}$$

where s stands for $A's$ market share and $(1-s)$ stands for $B's$ market share. In this model where the urban networks offer indistinguishable services, the market shares are taken to be exogenous. This is a relatively standard assumption, see for instance Laffont et.al. (2001). The profits of the rural network are given by

$$\begin{aligned}\Pi_R &= \gamma(p_R - c_R^T) + s(1-r_U)(t_{RA} - c_R^T) - s(1-r_R)\gamma(t_{AR} - c_R^T) \\ &\quad +(1-s)(1-r_U)(t_{RB} - c_R^T) - (1-s)(1-r_R)\gamma(t_{BR} - c_R^T) - c_R^F.\end{aligned}$$

Suppose that the urban networks are identical, i.e. $c_A^O = c_B^O > 0$, $c_A^T = c_B^T > 0$ and $c_A^F = c_B^F = 0$, (that is no fixed costs). We will examine symmetric equilibria, where

$$\begin{aligned}p_A &= p_B,\ p_A^B = p_B^A,\ p_A^R = p_B^R,\ t_A = t_B \text{ and} \\ t_{AR} &= t_{BR},\ t_{RA} = t_{RB}.\end{aligned}$$

As before, we will assume that $v > c_R^T$ and that $v > c_A^T$, and $c_R^T \geq c_A^T$.

2.2.2 Determination of Prices and Access fees.

Stage 1: Networks negotiate over access fees.

Stage 2: Given the outcome of negotiations, networks choose their prices.

We assume that first the networks negotiate pairwise over access fees and given these access fees, choose their prices non-cooperatively. We use the Nash-Bargaining solution to determine the outcome of the negotiations between two networks. Negotiations between two networks take place simultaneously and independently of all other negotiations. When two networks negotiate with each other, their disagreement payoffs are determined assuming that all other negotiatins have been terminated successfully.

Stage 2:

Price competition in urban market drives prices to marginal cost.

Proposition 5 *There exist a symmetric equilibrium where the urban networks set the price of calls equal to their perceived marginal cost, that is*

$$\begin{aligned}p_A &= c_A^O + c_A^T \\ p_A^B &= c_A^O + t_B \\ p_A^R &= c_A^O + t_{RA},\end{aligned}$$

The pricing behavior of B is analogous.

Proof. The result follows using the standard arguments of Bertrand competition. ■

The rural network is a monopolist so its sets

$$p_R = v_R = p_R^D.$$

Substituting the equilibrium prices in the expressions of profits we would get

$$\begin{aligned}\Pi_A &= \underbrace{s(1-s)r_u(t_A - c_A^T)}_{\text{net access revenue from the urban market}} + \underbrace{s(1-r_R)\gamma(t_{AR} - c_A^T)}_{\text{net access revenue from the rural market}} \\ \Pi_B &= \underbrace{s(1-s)r_u(t_B - c_B^T)}_{\text{net access revenue from the urban market}} + \underbrace{(1-s)(1-r_R)\gamma(t_{BR} - c_B^T)}_{\text{net access revenue from the rural market}}\end{aligned}$$

and the profits of the rural network are given by:

$$\begin{aligned}\Pi_R &= \gamma(v_R - c_R^T - c_R^0) + s(1-r_U)(t_{RA} - c_R^T) - s(1-r_R)\gamma(t_{AR} - c_R^T) \\ &\quad +(1-s)(1-r_U)(t_{RB} - c_R^T) - (1-s)(1-r_R)\gamma(t_{BR} - c_R^T) - c_R^F.\end{aligned}$$

Stage 1:

Now we study negotiations over access fees between the urban networks A and B and between an urban network and the rural network.

Negotiations between A and B :

The payoffs that accrue to these networks in case of agreement are given by

$$\begin{aligned}\Pi_A &= s(1-s)r_u(t_A - c_A^T) + s(1-r_R)\gamma(t_{AR} - c_A^T) \\ \Pi_B &= s(1-s)r_u(t_B - c_B^T) + (1-s)(1-r_R)\gamma(t_{BR} - c_B^T)\end{aligned}$$

and in case of disagreement- (since we are looking at a symmetric equilibrium we will assume that the market shares of the urban networks do not change in the event that they fail to sign an interconnection agreement.)

$$\begin{aligned}\Pi_A^{DAB} &= s(1-r_R)\gamma(t_{AR} - c_A^T) \\ \Pi_B^{DAB} &= (1-s)(1-r_R)\gamma(t_{BR} - c_B^T)\end{aligned}$$

$$\begin{aligned}t &\in \arg\max(\Pi_A - \Pi_A^{DAR})(\Pi_B - \Pi_B^{DAB}) \\ t &\in \arg\max[s(1-s)r_u(t - c_A^T)][s(1-s)r_u(t - c_B^T)]\end{aligned}$$

Proposition 6 *At a symmetric equilibrium the reciprocal access fee chosen by the urban networks is given by*

$$t = v - c^o \tag{7}$$

Proof. Since networks are symmetric, that is $c_A^T = c_B^T = c^T$; $c_A^o = c_B^o = c^o$; and because $s(1-s)r_U$ is a constant, the problem simplifes to:

$$t \in \arg\max(t - c^T)^2$$

This is maximized clearly by setting t equal to its largest possible value which is the one such that

$$v - t - c^0 \geq 0$$

hence

$$t = v - c^o.$$

■■

Negotiations between A and R :

$$(t_{AR}, t_{RA}) \in \arg\max(\Pi_A - \Pi_A^{DAR})(\Pi_R - \Pi_R^{DAR})$$

The first order condition of the Nash-Bargaining solution is given by

$$\Pi_A - \Pi_A^{DAR} = \Pi_R - \Pi_R^{DAR}. \tag{8}$$

In order to specify Π_A^{DAR} we need to find what the market share of A will be in the urban market in the case it disagrees with R. This is done in the Proposition that follows.

Proposition 7 *In the event that A fails to sign an interconnection agreement with R, then B will capture all the market.*

Proof. To see this note that the benefit that accrues to a customer from being connected with A in the case it fails to sign an interconnection agreement with R is given by

$$u_A = r_U s(v - p_A) + r_U(1-s)(v - p_A^B);$$

on the other hand the utility of a consumer connected to B is given by

$$u_B = r_U(1-s)(v-p_B) + r_U s(v-p_B^A) + (1-r_U)(v-p_B^R)$$

perfect comeption for rural phonecalls leads to an equilibrium where $p_A = p_B$ and $p_A^B = p_B^A$. Now if p_B^R is even ε below v all consumers prefer to be connected to network B. ■

Hence if A fails to sign an interconnection agreement with R it looses all its market share and its disagreement payoff is given by $\Pi_A^{DAR} = 0$. From the previous proposition it also follows that in case the negotiations between R and A fail, in this extreme case of competition, the profits to the rural network will remain unchanged.

$$\Pi_R^{DAR} = \Pi_R.$$

$$\Pi_A - \Pi_A^{DAR} = s(1-s)r_u(t_A - c_A^T) + s(1-r_R)\gamma(t_{AR} - c_A^T)$$

$$\Pi_R - \Pi_R^{DAR} = 0$$

From the FOC of the Nash-Bargaining solution we get that

$$s(1-s)r_u(t_A - c_A^T) + s(1-r_R)\gamma(t_{AR} - c_A^T) = 0$$

which gives us that

$$\begin{aligned} s(1-r_R)\gamma t_{AR} &= -s(1-s)r_u(t_A - c_A^T) + c_A^T s(1-r_R)\gamma \\ t_{AR} &= c_A^T - \frac{s(1-s)r_u(t_A - c_A^T)}{s(1-r_R)\gamma} \end{aligned}$$

and by substituting t_A, given by (7), we obtain

$$t_{AR} = c_A^T - \frac{s(1-s)r_u(v - c^o - c_A^T)}{s(1-r_R)\gamma}$$

Observe that in this environment one cannot determine t_{AR} and t_{RA} both simultaneously. We parametrize the solution by t_{RA}. There is actually an indeterminacy in the level of t_{RA} the highest possible value is

$$t_{RA} = v - c^o.$$

Since we are looking at a symmetric equilibrium, this must also be the outcome of negotiations between network B and network R. The proposition that follows summarizes the findings.

Proposition 8 *At a symmetric equilibrium, the access fee that the rural network pays to an urban network is given by*

$$t_{AR} = t_{BR} = c^T - \frac{s(1-s)r_U(v - c^o - c_A^T)}{s(1-r_R)\gamma}$$

and the access fee that the urban network pays to an rural network is given by

$$t_{RA} = t_{RB} = v - c^o.$$

It is worth observing that the access fee that the rural network pays to an urban network in below the cost of terminating the call. On the other hand the access fee that an urban network pays to the rural network is equal to the value of the phone call minus the cost of originating the call. That is, via the access fee the rural network can extract all the surplus - net of originating cost - that is generated from interconnection. The reason for this is the extremely strong bargaining power that it has. This power comes from the fact that R is the monopolist in selling access to rural customers. This together with the fierce competition that the urban networks face, allows R to achieve extremely favorable interconnection agreements.

Proposition 9 *At a symmetric equilibrium, the profits of the networks are give by*

$$\Pi_R = \gamma(v_R - c_R^T - c_R^0) + (1-r_U)(v - c^o - c_R^T) + s(1-s)r_u(v - c^o - c^T) + (1-r_R)\gamma\left(c_R^T - c^T\right) - c_R^F$$

$$\Pi_B = \Pi_A = 0.$$

Moreover the rural network's profits are greater compared to the ones without interconnection.

Proof. Hence the rural network's equilibrium profits are given by

$$\begin{aligned}
\Pi_R &= \gamma(v_R - c_R^T - c_R^0) + (1-r_U)(v - c^o - c_R^T) - (1-r_R)\gamma\left(c^T - \frac{s(1-s)r_u(v - c^o - c_A^T)}{s(1-r_R)\gamma} - c_R^T\right) - c_R^F \\
&= \gamma(v_R - c_R^T - c_R^0) + (1-r_U)(v - c^o - c_R^T) + (1-r_R)\gamma\left(\frac{s(1-s)r_u(v - c^o - c_A^T)}{s(1-r_R)\gamma} + c_R^T - c^T\right) - c_R^F \\
&= \gamma(v_R - c_R^T - c_R^0) + (1-r_U)(v - c^o - c_R^T) + s(1-s)r_u(v - c^o - c^T) + (1-r_R)\gamma\left(c_R^T - c^T\right) - c_R^F
\end{aligned}$$

It is easy to see that the rural network's profits without interconnection are

$$\Pi_R^{NI} = \gamma(v_R - c_R^T - c_R^0) - c_R^F.$$

Hence

$$\begin{aligned}
\Pi_R - \Pi_R^{NI} &= (1-r_U)(v - c^o - c_R^T) + (1-r_R)\gamma\left(\frac{s(1-s)r_u(v - c^o - c_A^T)}{s(1-r_R)\gamma} + c_R^T - c^T\right) > 0 \\
\text{since } v &> c^o + c_R^T \text{ and } c_R^T > c^T.
\end{aligned}$$

Which is strictly greater than the rural network's profits with interconnection. Now the payoffs that accrue to an urban network are given by

$$\begin{aligned}\Pi &= \Pi_A = \Pi_B = -c^F + s(1-s)r_u(v - c^o - c^T) + s(1-r_R)\gamma\left(c^T - \frac{s(1-s)r_u(v-c^o-c^T_A)}{s(1-r_R)\gamma} - c^T\right) \\ &= s(1-s)r_u(v-c^o-c^T) - \left(s(1-s)r_u(v-c^o-c^T_A)\right) = 0\end{aligned}$$

■

Proposition 10 *The rural network's equilibrium profits are decreasing in r_R and in r_U.*

Proof. Recall that

$$\Pi_R = \gamma(v_R - c^T_R - c^0_R) + (1-r_U)(v - c^o - c^T_R) + s(1-s)r_u(v-c^o-c^T) + (1-r_R)\gamma\left(c^T_R - c^T\right) - c^F_R$$

Then

$$\begin{aligned}\frac{\partial \Pi_R}{\partial r_U} &= -(v-c^o-c^T_R) + s(1-s)(v-c^o-c^T) \\ &= -(v-c^o)(1+s^2-s) - s(1-s)c^T < 0 \\ \text{since } v &> c^o \\ \frac{\partial \Pi_R}{\partial r_R} &= -\gamma\left(c^T_R - c^T\right) < 0.\end{aligned}$$

■

The results in this section demonstrate that even the fiercer form of competition does not eliminate the possibility of cross-subsidization across markets. But in order to obtain a yet more complete picture of the mechanisms we will later examine the, maybe, more realistic case where networks have some market power in variable demand model.

3 The Variable Demand Model

3.1 Monopoly in the Urban Market

Here we will expand our analysis to allow for elastic demand. This modeling will also allow for non-trivial analysis of the socially optimal level of prices and access fees.

Demand Structure in the Urban Market

Suppose that the utility from consuming telecommunication services by a representative consumer is

$$u(q) = (a_U - 0.5q)q + m.$$

As usual, given income I, a consumer seeks to maximize her utility subject to the budget constraint

$$p_i q_i + m \leq I.$$

Suppose that a consumer has joined network i, then given price p_i, the quantity that maximizes her utility is given by

$$q(p_i) = a_U - p_i,$$

and her indirect utility

$$v(p_i) = \frac{(a_U - p_i)^2}{2}.$$

There a measure one of such consumers, so aggregate demand is given by

$$q(p_i) = a_U - p.$$

Demand Structure in the Rural Market

In the rural market the local network faces a measure $\gamma \in [0, 1]$ of identical consumers. We assume that the aggregate demand function is given by

$$q_R(p_R) = \gamma(a_R - p_R).$$

Cost Structure and Profits

This is all exactly as in the unit demand model, with the difference that q is now a function of price. As before we define

$$\begin{aligned} \pi_i(p) &= q(p)\left(p - c_A^o - c_A^T\right), \text{ and} \\ \pi_R(p) &= q_R(p)\left(p - c_R^o - c_R^T\right) - c_R^F, \end{aligned}$$

then $A's$ profits can be rewritten as

$$\Pi_A = (a_U - p_A)\left(p_A - c_A^o - c_A^T\right) + \left[(1 - r_R)\left(t_{AR} - c_A^T\right)\gamma(a_R - p_R) - (1 - r_U)\left(t_{RA} - c_A^T\right)(a_U - p_A)\right],$$

or

$$\Pi_A = \underbrace{\pi_A(p_A)}_{\text{profits from retail}} + \underbrace{\left[(1 - r_R)\left(t_{AR} - c_A^T\right)q_R(p_R) - (1 - r_U)\left(t_{RA} - c_A^T\right)q(p_A)\right]}_{\text{net access revenue from rural market}} - c_A^F.$$

3.1.1 Ramsey Pricing

We would like to investigate what prices and access fees maximize welfare subject to the constraint that none of the networks, that is the urban and the rural network, is running a loss. Let λ_1 and λ_2 denote the Lagrangian multipliers associated with these constraints. Welfare is given by

$$\begin{aligned}
W \quad = \quad & \underbrace{\frac{(a_U - p_A)^2}{2}}_{\text{consumer surplus in urban market}} + \underbrace{\frac{\gamma(a_R - p_R)^2}{2}}_{\text{consumer surplus in rural market}} \\
& + \underbrace{(1+\lambda_U)\left(\pi_A(p_A) + \left[(1-r_R)\left(t_{AR} - c_A^T\right)\gamma(a_R - p_R) - (1-r_U)\left(t_{RA} - c_A^T\right)(a_U - p_A)\right]\right)}_{\text{producer surplus in urban market}} \\
& + \underbrace{(1+\lambda_R)\left(\pi_R(p_R) - c_R^F + \left[(1-r_U)\left(t_{RA} - c_R^T\right)(a_U - p_A) - (1-r_R)\left(t_{AR} - c_R^T\right)\gamma(a_R - p_R)\right]\right)}_{\text{producer surplus in rural market}}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial W}{\partial p_U} &= -(a_U - p_A) + (1+\lambda_U)\left((a_U - p_A) - \left(p_A - c_A^o - c_A^T\right) + (1-r_U)\left(t_{RA} - c_A^T\right)\right) \\
&\quad -(1+\lambda_R)(1-r_U)\left(t_{RA} - c_R^T\right) \\
&= \lambda_U(a_U - p_A) + (1+\lambda_U)\left((1-r_U)\left(t_{RA} - c_A^T\right) - \left(p_A - c_A^o - c_A^T\right)\right) - (1+\lambda_R)(1-r_U)\left(t_{RA} - c_R^T\right) \\
&= \lambda_U a_U + (1+\lambda_U)\left((1-r_U)\left(t_{RA} - c_A^T\right) + c_A^o + c_A^T\right) - (1+\lambda_R)(1-r_U)\left(t_{RA} - c_R^T\right) - (1+2\lambda_U)p_A = 0
\end{aligned}$$

which gives

$$p_A = \frac{\lambda_U a_U + (1+\lambda_U)\left((1-r_U)\left(t_{RA} - c_A^T\right) + c_A^o + c_A^T\right) - (1+\lambda_R)(1-r_U)\left(t_{RA} - c_R^T\right)}{(1+2\lambda_U)}$$

$$\begin{aligned}
\frac{\partial W}{\partial p_R} &= -\gamma(a_R - p_R) - (1+\lambda_U)(1-r_R)\left(t_{AR} - c_A^T\right)\gamma \\
&\quad +(1+\lambda_R)\left(\gamma(a_R - p_R) - \gamma p_R + \gamma c_R^o + \gamma c_R^T - c_R^F + (1-r_R)\left(t_{AR} - c_R^T\right)\gamma\right) \\
&= \lambda_R\gamma(a_R - p_R) - (1+\lambda_U)(1-r_R)\left(t_{AR} - c_A^T\right)\gamma + (1+\lambda_R)\left(-\gamma p_R + \gamma c_R^o + \gamma c_R^T - c_R^F + (1-r_R)\left(t_{AR} - c_R^T\right)\gamma\right. \\
&= \lambda_R\gamma a_R - (1+\lambda_U)(1-r_R)\left(t_{AR} - c_A^T\right)\gamma + (1+\lambda_R)\left(\gamma c_R^o + \gamma c_R^T - c_R^F + (1-r_R)\left(t_{AR} - c_R^T\right)\gamma\right) - (1+2\lambda_R)p
\end{aligned}$$

$$p_R = \frac{\lambda_R\gamma a_R - (1+\lambda_U)(1-r_R)\left(t_{AR} - c_A^T\right)\gamma + (1+\lambda_R)\left(\gamma c_R^o + \gamma c_R^T - c_R^F + (1-r_R)\left(t_{AR} - c_R^T\right)\gamma\right)}{(1+2\lambda_R)}$$

Now let us look for the welfare maximizing access fees.

$$\begin{aligned}
\frac{\partial W}{\partial t_{AR}} &= (1+\lambda_U)\left((1-r_R)\gamma(a_R - p_R)\right) - (1+\lambda_R)\left((1-r_R)\gamma(a_R - p_R)\right) \\
\frac{\partial W}{\partial t_{RA}} &= -(1+\lambda_U)(1-r_U)(a_U - p_A) + (1+\lambda_R)(1-r_U)(a_U - p_A)
\end{aligned}$$

If

$$\frac{\partial W}{\partial t_{AR}} > 0 \text{ then } t_{AR} \text{ as large as possible}$$
$$\frac{\partial W}{\partial t_{AR}} < 0 \text{ then } t_{AR} \text{ as small as possible}$$

3.2 Horizontal Differentiation in the Urban Market: The Case of Variable Demand

We consider three telecommunication networks, A, B and R. Networks A and B operate in the same geographic area, the low cost area (or urban area) and are competing for subscribers. Networks A and B are horizontally differentiated and each can provide full coverage of the urban market. Consumers are uniformly located on the segment [0,1] and each network is located at the ends of this line, $x_A = 0$ and $x_B = 1$. Network R is located in the high cost area, the rural area, and is not competing for customers with networks A and B. When all networks are interconnected, then a customer of a network i can place a phone call to a customer of network j, where $i, j = A, B, R$.

This is a model that captures the state of mature competition of the industry. There are a number of interconnected networks where some compete directly for customers and others not, but all firms need to purchase necessary inputs from each other. Each consumer can subscribe to one network only. When a subscriber of B calls a subscriber of A, B has to 'buy' access to A's subscribers. In short, network i sells access to network j and the opposite. Hence there is 'two-way' access with competition between network A and network B and 'two-way' access without competition between network R and network $i = A, B$.

- **Demand Structure in the Urban Market**[6]

In the **urban** market networks A and B compete for customers. An individual's utility from being a subscriber to network i depends on the degree of interconnection of network i with other networks, S_i, the price per phone call, p_i, and the other extra features network i offers, β_i.

We assume that

- consumers derive utility only from placing and not from receiving phone calls
- Balanced calling pattern. Within the urban market a consumer calls with equal probability a consumer in the same network and one belonging to a different network.[7]

[6] The demand structure resembles the one in Carter and Wright (1999). The analysis in Economides, Lopomo and Wright (1996) provides a justification for it.

[7] For further discussion on the balanced calling pattern see Laffont et al. 1998a.

- no price discrimination between phone calls completed within a network, and phone calls across networks.

We follow Carter and Wright (1999) in the specification of consumer preferences. We assume that consumers have additively separable preferences between phone calls and extra features (the latter is where the networks differ). To an individual the value of being interconnected to a network depends on the fraction of consumers that she can reach by joining it, that is, on the size of the network she joins, S_i. An urban network's size, S_i, $i = A, B$ depends on the interconnection agreements it has signed with the other networks, and on its market share (which depends on its price and its rivals price). We will later calculate the size of each network under various interconnection scenarios.

We assume that marginal utility is linear in the total quantity of phone calls, more specifically

$$u(q) = (a - 0.5q)q,$$

Note that in this model q_i denotes the aggregate quantity of phone calls that the average subscriber of network i plans to make. Some of these phone calls are completed within the urban market , Q_U^i, whereas some are made to customers in the rural market, Q_R^i, that is $q_i = Q_R^i + Q_U^i$. We assume that $Q_U^i = r_U q_i$ and $Q_R^i = (1 - r_U)q_i$, where r_U denotes the percentage of total phone calls that are completed within the urban market.[8]

As usual, given income I, a consumer seeks to maximize her utility subject to the budget constraint

$$p_i q_i + m \leq I.$$

Suppose that a consumer has joined network i, then given price p_i, the quantity that maximizes her utility is given by

$$q(p_i) = a - p_i,$$

and her indirect utility

$$\psi(p_i, S_i, \beta_i, I) = v(p_i)S_i + \beta_i + m$$

[8] If a representative consumers' preferences are represented by

$$u(Q_R, Q_U) = Q_U^{r_U} Q_R^{(1-r_U)}$$

where Q_R denotes the calls within the urban market and Q_U the ones directed to the rural market, and $P_U = P_R$, and I denotes the income of a representative consumer, then demands are

$$\begin{aligned} Q_U &= r_U Q \\ Q_R &= (1 - r_U)Q, \end{aligned}$$

where $Q = \frac{I}{P}$, which is the sum of $Q_U + Q_R$.

where

$$v(p_i) = \frac{(a - p_i)^2}{2};$$

m represents the consumption of other goods, β_i stands for the extras associated with being connected to network i, and S_i is the degree of interconnection of network i, $i = A, B$.

- **Demand Structure in the Rural Market**

In the rural market the local network faces a unit measure of identical consumers.[9] Since there is only one network in the rural market, we will not model explicitly the utility that an individual enjoys by being connected to the network. We will focus on the aggregate demand function, which we will assume for simplicity that it is given by

$$q_R(p_R) = \gamma(a - p_R),$$

where $\gamma \in [0, 1]$ and $q_R(p_R)$ denotes the aggregate quantity of phone calls that the average subscriber of the rural network plans to make at price p_R. The total quantity of phone calls equals the sum of calls directed to urban customers, Q_U^R, and calls directed to rural consumers, Q_R^R, that is $q_R = Q_R^R + Q_U^R$. We assume that $Q_U^R = (1 - r_R)q_R$ and $Q_R^R = r_R q_R$, where r_R is the fraction of total calls made by the average rural customer that are completed within the rural market.

We now determine the size of network i, S_i under different degrees of interconnection.

- **Size of the networks**

Table (1) contains the size of a network for different cases of interconnection agreements taking the market shares of A and B as given. We use s to denote $A's$ market share in case all networks have signed interconnection agreements, s_1 is $A's$ share when negotiations between A and B fail, s_2 is $A's$ market share when negotiations between A and R fail, and finally s_3 is $A's$ share when negotiations between B and R fail. If A has signed interconnection agreements with B and R, then $S_A = 1$, and a subscriber of network A can reach a subscriber j irrespective of the network that j belongs. If it has signed an interconnection agreement only with B then $S_A = r_U$, (the rural customers cannot be reached by a subscriber of A), and finally if it has signed an interconnection agreement with R only, $S_A = r_U s + (1 - r_U)$, where s denotes $A's$ market share.

- **Market shares when the urban networks are interconnected to each other:** $S_A = S_B = 1$.

[9] Or alternatively, one representative consumer.

Table 1: The Size of Networks

	Size of **A**	Size of **B**	Size of **R**
All agree	$S_A = 1$	$S_B = 1$	$S_R = 1$
A, **B**, disagree	S_A^{DAB} $= r_U s_1 + (1 - r_U)$	S_B^{DAB} $r_U(1 - s_1) + (1 - r_U)$	$S_R^{DAB} = 1$
A, **R** disagree	$S_A^{DAR} = r_U$	$S_B^{DAR} = 1$	$S_R^{DAR} =$ $r_R + (1 - r_R)(1 - s_2)$
B, **R** disagree	$S_A^{DBR} = 1$	$S_A^{DBR} = r_U$	$S_R^{DBR} =$ $r_R + (1 - r_R)s_3$

,

Now we determine the market share of an urban network for various degrees of interconnection with the other networks. If all networks are interconnected, that is $S_i = 1$, for $i = A, B, R$, a consumer's decision on which network to join, depends on p_A, p_B and β_A and β_B. For a consumer located at x, $\beta_A = (1 - x)\alpha$ and $\beta_B = x\alpha$. To be more specific, if

$$\begin{aligned} \psi(p_A, S_A, \beta_A, I) &\geq \psi(p_B, S_B, \beta_B, I) \text{ or} \\ v(p_A) + (1 - x)\alpha + m &\geq v(p_B) + x\alpha + m \end{aligned}$$

then the consumer located at x will subscribe to network A. With interconnection, the market share of network A is given by

$$s(p_A, p_B) = \begin{cases} 0 \text{ if } x < 0 \\ 1 \text{ if } x > 1 \\ x \text{ otherwise} \end{cases} \tag{9}$$

where

$$x(p_A, p_B) = \frac{1}{2} + \frac{v(p_A) - v(p_B)}{2\alpha}$$

In a symmetric equilibrium $s(p, p) = \frac{1}{2}$ and $\frac{\partial s(p,p)}{\partial p_A} = \frac{-q(p)}{2\alpha}$, which will be used in later derivations.

- **Market shares when A and B are not interconnected:** $S_A = (r_U s_1 + (1 - r_U))$ **and** $S_B = (r_U(1 - s_1) + (1 - r_U))$

In the case that the urban networks are not interconnected, the net surplus for a consumer located at x, from being connected to network A, when A and B failed to sign an interconnection agreement is given by

$$v(p_A)(r_U s_1 + (1-r_U)) + (1-x)\alpha,$$

and the net surplus from being connected to network B is given by

$$v(p_B)(r_U(1-s_1) + (1-r_U)) + \alpha x,$$

where s_1 stands for network $A's$ market share, for the case that urban firms fail to reach an agreement, and p_i denotes the price of network i, $i = A, B$. With no interconnection between A and B, network $A's$ market share is given by

$$s_1 = \begin{cases} 0 \text{ if } x_1 < 0 \\ 1 \text{ if } x_1 > 1 \\ x_1 \text{ otherwise} \end{cases} \tag{10}$$

where

$$x_1(p_A, p_B) = \frac{\alpha - v(p_B) + v(p_A)(1-r_U)}{2\alpha - (v(p_A) + v(p_B))r_U}. \tag{11}$$

Network $B's$ market share is given by $1 - s_{1.}$ In a symmetric equilibrium, $p_A = p_B$ and $s_1 = \frac{1}{2}$ and $\frac{\partial s_1(p,p)}{\partial p_A} = \frac{(a-p)(2-r_U)}{4(\alpha - v(p))}$.

- **Market shares when A and R are not interconnected: $S_A = r_U$ and $S_B = 1$**

When A and R fail to sign an interconnection agreement, the net surplus for a consumer located at x, is given by

$$v(p_A)r_U + (1-x)\alpha + m,$$

and the net surplus from being connected to network B is given by

$$v(p_B) + \alpha x + m.$$

With no interconnection between A and R, network $A's$ market share is given by

$$s_2 = \begin{cases} 0 \text{ if } x_2 < 0 \\ 1 \text{ if } x_2 > 1 \\ x_2 \text{ otherwise} \end{cases} \tag{12}$$

where

$$x_2(p_A, p_B) = \frac{1}{2} + \frac{v(p_A)r_U - v(p_B)}{2\alpha}. \tag{13}$$

Network $B's$ market share is given by $1 - s_2$.

- **Market shares when B and R are not interconnected: $S_A = 1$ and $S_B = r_U$**

Using the, by now familiar, procedure, we get that with no interconnection between B and R, network $A's$ market share is given by

$$s_3 = \begin{cases} 0 \text{ if } x_3 < 0 \\ 1 \text{ if } x_3 > 1 \\ x_3 \text{ otherwise} \end{cases}$$

where

$$x_3(p_A, p_B) = \frac{1}{2} + \frac{v(p_A) - v(p_B) r_U}{2\alpha}. \tag{14}$$

Network $B's$ market share is given by $1 - s_3$.

- **Cost Structure and Profits**

Now we specify the cost structure of the networks. Let[10] c_i^o denote the cost of originating a phone call from network i, and c_i^T the cost of terminating a phone call at network i, $i = A, B, R$. Network i incurs a fixed cost c_i^F of servicing a customer. Also let $s \in [0,1]$ denote network $A's$ market share and $1 - s$, $B's$ market share. That is, we implicitly assume that consumer preferences are such that all of them wish to purchase telecommunication services.

We use p_i to denote the price set by network i and t_i to designate the access fee network j pays to network i, each time a subscriber of network j calls a subscriber of network $i = A, B$ and $j = A, B$, $i \neq j$. We also use t_{iR} , (t_{Ri}), to denote the access fee that the rural network $R, (i)$, pays to network i, (R), each time a subscriber of network R, (i), calls a subscriber of network i, (R), where $i = A, B$. Networks do not charge different prices for phone calls completed within the network and phone calls completed across networks. Network $A's$ profits are given by

$$\begin{aligned} \Pi_A &= sq(p_A) \left\{ p_A - c_A^0 - \left[s r_u c_A^T + (1-s) r_u t_B + (1-r_u) t_{RA} \right] \right\} - s c_A^F + \\ &\quad s(1-s) r_u (t_A - c_A^T) q(p_B) + s(1 - r_R)(t_{AR} - c_A^T) q_R(p_R) \end{aligned}$$

Let

$$\begin{aligned} \pi_i(p) &= q(p) \left(p - c_i^o - c_i^T \right) - c_i^F, \; i = A, B \text{ and} \\ \pi_R(p) &= q_R(p) \left(p - c_R^o - c_R^T \right) - c_R^F, \end{aligned}$$

then $A's$ profits can be rewritten as

$$\begin{aligned} \Pi_A &= s\pi_A(p_A) + s(1-s) r_u \left[\left(t_A - c_A^T \right) q(p_B) - \left(t_B - c_A^T \right) q(p_A) \right] \\ &\quad + s \left[(1 - r_R) \left(t_{AR} - c_A^T \right) q_R(p_R) - (1 - r_u) \left(t_{RA} - c_A^T \right) q(p_A) \right]. \end{aligned} \tag{15}$$

[10] The cost structure is similar to the one in Armstrong (1998).

The first term in the above expression is firm $A's$ profit in the retail sector; the second term is the firm's profit from buying or selling access in the urban market; and the third term is the firm's profit from buying or selling access in the rural market. Similarly the profits of network B are given by

$$\begin{aligned} \Pi_B &= (1-s)\pi_B(p_B) + s(1-s)r_u\left[\left(t_B - c_B^T\right)q(p_A) - \left(t_A - c_B^T\right)q(p_B)\right] \\ &\quad +(1-s)\left[(1-r_R)\left(t_{BR} - c_B^T\right)q_R(p_R) - (1-r_u)\left(t_{RB} - c_B^T\right)q(p_B)\right]. \end{aligned} \tag{16}$$

Finally the profits of the network serving the rural area are given by

$$\begin{aligned} \Pi_R &= \pi_R(p_R) + s\left[(1-r_u)\left(t_{RA} - c_R^T\right)q(p_A) - (1-r_R)\left(t_{AR} - c_R^T\right)q_R(p_R)\right] \\ &\quad +(1-s)\left[(1-r_u)\left(t_{RB} - c_R^T\right)q(p_B) - (1-r_R)\left(t_{BR} - c_R^T\right)q_R(p_R)\right]. \end{aligned} \tag{17}$$

Determination of Prices and Access fees

There are three players in this game: two urban networks denoted by A and B and a rural network, R. We look at a scenario where networks negotiate over interconnection agreements and choose their prices non-cooperatively. The US Telecommunication Act of 1996 states that access fees should be negotiated among networks subject to regulatory approval. In Europe interconnection agreements should be negotiated within the framework of the European Law and the supervision of the National Regulatory Agencies. In other countries, for instance in New Zealand, after the privatization of the national telecommunication provider, there is no regulatory agency and networks should reach interconnection agreements that do not violate the existing antitrust laws. We examine the determination of access fees through free negotiations without regulation. Networks will be assumed to negotiate pairwise over access fees. The outcome of negotiations is determined by the Nash Bargaining Solution.

The Nash Bargaining solution is a cooperative solution concept, that allows for a parsimonious representation of the conflicts of interests in a given negotiation. According to the Nash Bargaining Solution, negotiating parties split the 'gains from reaching an agreement', equally among each other. For a discussion of the application of the Nash Bargaining Solution to various bargaining problems and its relationship with non-cooperative dynamic solution concepts see Binmore, Rubinstein and Wolinsky (1986).

We will first look at the case that prices and access fees are determined sequentially.

•**Assumption TC:** 'Transportation cost' α is high enough to ensure that an equilibrium exists.

3.2.1 A. Sequential-Move Game

We will study the equilibrium choices of prices and access fees, in a game that consists of the following stages.

Stage 1: In stage 1 each urban network negotiates access fees with the rural network: (t_{iR}, t_{Ri}), $i = A, B$.

Stage 2: (a)The urban networks negotiate over access fees (t_A, t_B) and (b) given the access fees choose their prices (p_A, p_B).

Stage 3: The rural network decides whether to operate or not and sets its price, p_R.

Stage 4: Consumption takes place.

We derive a subgame perfect equilibrium of the game. We will assume that the urban networks are symmetric, that is $c_A^o = c_B^o = c^o$, $c_A^T = c_B^T = c^T$ and $c_A^F = c_B^F = c^F$ and we will restrict attention to symmetric equilibria.[11] At a symmetric equilibrium it holds that $t_A = t_B = t$ and $p_A = p_B = p$, $t_{AR} = t_{BR} = t_{UR}$ and $t_{RA} = t_{RB} = t_{RU}$. Reciprocal access fees between relatively homogeneous networks, like network A and B in our model, are indeed encouraged by many governments, see Carter and Wright (1999), Laffont et al. (1998a).

- **Stage 3: The rural network chooses a price.**

In order to derive the subgame perfect equilibrium of this game, one has to specify what price a network will choose after each history of the game. In other words, for the subgame starting at stage 3, a network's strategy has to describe a price choice for each contingency. We need to specify the price that a network will choose for the case that networks i and j reach an agreement as well as for the case that negotiations between i and j fail, where $i, j = A, B, R$ and $i \neq j$.

Suppose that all networks have signed interconnection agreements. Then, given the vector of negotiated fees (t_{RU}, t_{UR}, t), and the price chosen by the urban networks p, the rural network chooses the price that maximizes its profits. At a symmetric equilibrium the rural network's profits are given by

$$\begin{aligned} \Pi_R &= \pi_R(p_R) + \\ & \left[(1 - r_u)\left(t_{RU} - c_R^T\right) q(p) - (1 - r_R)\left(t_{UR} - c_R^T\right) \gamma q(p_R)\right] \end{aligned} \tag{18}$$

Assuming an interior solution, the first order conditions for profit maximization are given by

$$\frac{\partial \Pi_R}{\partial p_R} = \gamma(a - 2p_R + c_R^o - c_R^T) + \gamma(1 - r_R)(t_{UR} - c_R^T) = 0$$

[11] Asymmetric equilibria can be interesting in this model. I have computed examples (preliminary results) that show that there are equilibria, where one of the urban networks reaches such an agreement with the rural network that makes it unprofitable for the other network to reach an interconnection agreement with the rural market. The agreement is so favorable to the rural network that the competitor does not find profitable to much it. The coalition of a single urban network and the rural network enjoys higher profits compared to the symmetric equilibrium.

which gives

$$p_R = \frac{a + c_R^o + c_R^T + (t_{UR} - c_R^T)(1 - r_R)}{2}.$$

Using this straightforward procedure one can determine the price that the urban network will charge in different histories, when for instance negotiations between R and A fail.

- **Stage 2: Urban Networks Choose Access Fees and Prices**

Suppose that the urban networks have agreed on a reciprocal access fee t and the negotiations with the rural network have already taken place. Given t and (t_{UR}, t_{RU}), we derive a symmetric equilibrium of the price game between network A and network B. Consider network A; its profits are given by (15). Let us define $R_{AR}(p_R, p)$ by

$$R_{AR}(p_R, p) = (1 - r_R)\left(t_{UR} - c^T\right) q_R(p_R) - (1 - r_u)\left(t_{RU} - c^T\right) q(p).$$

This is $A's$ net revenue from selling access to R. Assume an interior solution. If p_A is a profit maximizing price then it satisfies

$$\begin{aligned} \frac{\partial \Pi_A(p,p)}{\partial p_A} &= \frac{\partial s(p,p)}{\partial p_A}\pi_A(p) + s(p,p)\frac{\partial \pi_A(p)}{\partial p_A} - s(1-s)r_u(t - c^T)\frac{\partial q(p)}{\partial p_A} \\ &+ \frac{\partial s(p,p)}{\partial p_A} R_{AR}(p_R, p) + s(p,p)\frac{\partial R_{AR}(p_R, p)}{\partial p_A} = 0. \end{aligned}$$

Access Fees: From the analysis in Armstrong (1998), Carter and Wright (1999) and Laffont, Rey and Tirole (1998a) we know that if an equilibrium exists, and access fees are required to be reciprocal, then in a deregulated duopoly market, networks will choose access fees that support prices that maximize joint profits. In our environment, where the urban networks are interconnected with the rural network, they will choose access fees that support the price that maximizes joint profit.

Joint profit for networks A and B are given by

$$\Pi_{A+B}(p) = \pi(p) + R_{AR}(p_R, p)$$

and assuming an interior solution, the FOC for joint profit maximization is given by

$$\frac{\partial \pi(p)}{\partial p} + \frac{\partial R_{AR}(p_R, p)}{\partial p} = 0. \qquad (19)$$

The price that maximizes joint profits is given by

$$p = \frac{a + c^o + c^T + (1 - r_U)(t_{RU} - c^T)}{2}. \qquad (20)$$

From (19) we get that the access fee that sustains joint profit maximization is given by[12]

$$t = c^T + \frac{2q(p)\,[\pi(p) + R_{AR}(p_R, p)]}{-\alpha r_u q'(p)}. \tag{21}$$

- **Stage 1: Negotiations between urban networks & rural networks over** (t_{UR}, t_{RU}).

Now, we turn on the analysis of the negotiations between an urban network, say network A, and the rural network. From the analysis of stage 2, we know that if network A is interconnected with network B and the rural network, for given t_{UR} and t_{RU}, network $A's$ profits are given by

$$\Pi_A = \frac{1}{2}\,[\pi(p) + R_{AR}(p_R, p)]$$

and in case network A disagrees with the rural network, its profits will be given by

$$\Pi_A^{DAR} = s_1 r_U \pi(p_A^{DAR}) + s_1(1-s_1) r_U[(t-c)q(p_B) - (t-c)q(p_A^{DAR})],$$

where s_1 denotes the market share of network A in case it disagrees with network R, and is given by (11). Also p_A^{DAR} stands for the price the rural network will charge network A in the case that negotiations between A and R collapse, (analytical expression of the optimal price in case of disagreement is very difficult to obtain). The agreement profits of the rural network are given by (18) and the disagreement payoffs are given by

$$\begin{aligned}\Pi_R^{DAR} &= (r_R + (1-s_1)(1-r_R))\pi_R(p_R^{DAR}) \\ &\quad + (1-s_1)\left[(1-r_U)\,(t_{RU}-c)(a-p) - (1-r_R)(t_{UR}-c)\gamma(a-p_R^{DAR})\right]\end{aligned}$$

the price of the rural network in case of disagreement is given by

$$p_R^{DAR} = \frac{r_R\gamma(a + c_R^o + c_R^T) + (1-r_R)(1-s_2)(\gamma(a + c_R^o + c_R^T) + (t_{UR} - c_T^R)\gamma)}{2}$$

As discussed earlier we use the Nash Bargaining Solution to obtain the outcome on negotiations between network A and the rural network. The solution is a pair (t_{RA}, t_{AR}) that maximizes

$$(t_{AR}, t_{RA}) \in \arg\max(\Pi_A - \Pi_A^{DAR})(\Pi_R - \Pi_R^{DAR}).$$

Since we are looking at a symmetric equilibrium negotiations, between R and B are identical to the negotiations between R and A.

We proceed by solving for the symmetric equilibrium of a parametrized example.

[12] Carter and Wright (1999) present an example where the access fee that sustains a price that maximizes joins profits, is equal to the one we would get if we used the Nash Bargaining Solution.

Table 2: Sequential-Moves: Symmetric Equilibrium: A

	$c_R^F = 9,\ r_R = 0.5,$ $\mathbf{r}_U = \mathbf{0.2}$	$c_R^F = 9,\ r_R = 0.5,$ $\mathbf{r}_U = \mathbf{0.6}$	$c_R^F = 9,\ r_R = 0.5,$ $\mathbf{r}_U = \mathbf{0.9}$
$p_A = p_B$	6.9058	6.4754	6.1333
p_R	6.1082	6.3392	6.4870
$p_A^{DAR} = p_B^{DBR}$	6.5581	5.7383	5.4590
$p_R^{DAR} = p_R^{DBR}$	6.3776	6.5550	6.6663
$t_A = t_B$	5.3791	3.5657	3.4106
Π_R	1.7032	-0.3580	-2.1852
$\Pi_R^{DAR} = \Pi_R^{DBR}$	-0.1954	-1.3257	-2.2409
$\Pi_A = \Pi_B$	3.5381	5.4595	7.0137
$\Pi_A^{DAR} = \Pi_B^{DBR}$	1.3801	4.4229	6.9580
$t_{RA} = t_{RB}$	3.2645	3.3771	3.6663
$t_{AR} = t_{BR}$	-1.5672	-0.6431	-0.0519

Suppose that

$$q(p) = 10 - p$$

$$c^o = c^T = 1,\ c^F = 0, c_R^o = c_R^T = 2$$

$$\gamma = 0.5,\ \alpha = 50$$

As noted earlier we solve for a symmetric equilibrium, and we do not impose any cross market price restrictions.

Discussion of the Results

Table 2 contains the equilibrium prices, access fees and profits that accrue to the three networks as we increase r_U. Recall that r_U denotes the fraction of total calls made by urban customers that are completed within the urban market. As r_U increases, the less value the urban customers place in being interconnected with the rural market. The equilibrium profits of the rural network decrease as r_U increases. Later we will verify analytically that this is also the case in alternative scenarios regarding the degree of competition in the urban market. In the above example the profits that would accrue to the rural network if it were not interconnected with the urban market, are given by $\pi_R = -4.5$, when $c_R^F = 9$ and $\pi_R = -0.5$, when

Table 3: Sequential-Moves: Symmetric Equilibrium: B

	$r_U = 0.4, r_R = 0.7, \mathbf{c}_R^F = \mathbf{9}$	$r_U = 0.4,\ r_R = 0.7, \mathbf{c}_R^F = \mathbf{5}$
$p_A = p_B$	6.6876	6.5734
p_R	6.3737	6.5734
$p_A^{DAR} = p_B^{DBR}$	6.0369	5.9295
$p_R^{DAR} = p_R^{DBR}$	6.6175	6.7420
$t_A = t_B$	4.0623	3.9589
Π_R	0.1427	3.0870
$\Pi_R^{DAR} = \Pi_R^{DBR}$	-1.4664	1.4093
$\Pi_A = \Pi_B$	4.6224	5.3969
$\Pi_A^{DAR} = \Pi_B^{DBR}$	2.8509	3.6226
$t_{RA} = t_{RB}$	3.2919	3.2936
$t_{AR} = t_{BR}$	-2.1757	-0.8438

$c_R^F = 5$. With interconnection the rural network's profits for fixed cost $c_R^F = 9$ can be $\Pi_R = 1.7$ compared to $\pi_R = -4.5$ without interconnection, (see the results in Table 2). Table 3.3 shows how the equilibrium prices, access fees and profits change as the fixed cost of the rural network falls from $c_R^F = 9$ to $c_R^F = 5$. Another interesting feature of the equilibrium, is that the access fee that the rural network pays to an urban network, whenever a rural customer places a call to an urban customer, t_{UR}, $U = A, B$, is *negative.* The urban network finds profitable to subsidize the rural network. The reason is that by reaching this agreement the urban network increases its revenues because urban customers want to place phone calls to the rural customers. These subsidies lead to access revenues for the rural network that may be high enough to cover the losses in the retail sector. In particular, observe that ceteris paribus, the subsidy decreases with r_U, (see Table 2). That is, the subsidies decrease as the percentage of phone calls that are completed within the urban market increases. If urban consumers like to call customers in the same geographic area, then the urban network does not value interconnection to the rural network as much as it would if urban customers tended to place a higher percentage of their total demand towards the rural market. Notice also that prices in the urban and the rural market do not differ substantially and there are sometimes even lower in the rural market.

3.2.2 B. Simultaneous-Moves

We assume an interior solution and derive the first order conditions that optimal prices must satisfy. Taking $(p_k, t_A, t_B, t_{AR}, t_{RA}, t_{BR}, t_{RB})$ as given, $k = A, B, R$ and $k \neq i$, network i chooses p_i that solves

$$\frac{\partial \Pi_i}{\partial p_i} = 0 \; . \tag{p$_i$}$$

Simultaneously with choosing prices, networks negotiate pairwise in order to determine interconnection *access fees*. In particular, network A negotiates with network B over t_A and t_B, and network R negotiates with A and B over the determination of t_{AR}, t_{RA} and t_{BR} and t_{RB}. All negotiations take place simultaneously and independently from one another, in other words, we assume that the outcome of the negotiations between R and A is not known to R and B when they bargain over (t_{BR}, t_{RB}).

- **Negotiations between A and B over** (t_A, t_B)

As noted earlier we will use the Nash Bargaining solution in order to determine the outcome of negotiations between A and B. The Nash Solution to this bargaining situation is a pair (t_A, t_B) such that

$$(t_A, t_B) \in \arg\max(\Pi_A - \Pi_A^{DAB})(\Pi_B - \Pi_B^{DAB}), \tag{22}$$

where Π_A and Π_B are given by (15) and (16) respectively. The disagreement payoffs of network A are given by

$$\begin{aligned} \Pi_A^{DAB} &= s_1(r_U s_1 + (1 - r_U))\pi_A(p_A) + \\ & \quad s_1 \left[(1 - r_R)\left(t_{AR} - c_A^T\right)\gamma q(p_R) - (1 - r_u)\left(t_{RA} - c_A^T\right) q(p_A)\right] \end{aligned}$$

where s_1 is given by (11). Similarly the disagreement payoffs of network B are given by

$$\begin{aligned} \Pi_B^{DAB} &= (1 - s_1)(r_U(1 - s_1) + (1 - r_U))\pi_B(p_B) + \\ & \quad (1 - s_1)\left[(1 - r_R)\left(t_{BR} - c_B^T\right)\gamma q(p_R) - (1 - r_u)\left(t_{RB} - c_B^T\right) q(p_B)\right] . \end{aligned}$$

The first order conditions of the problem described in (22) reduce to

$$\Pi_A - \Pi_A^{DAB} - \Pi_B + \Pi_B^{DAB} = 0. \tag{23}$$

- **Negotiations between U and R over** (t_{UR}, t_{RU}), $U = A, B$

Networks A and R bargain over the determination of (t_{AR}, t_{RA}). Again, we use the Nash Solution to determine the outcome of these negotiations. Hence

$$(t_{AR}, t_{RA}) \in \arg\max(\Pi_A - \Pi_A^{DAR})(\Pi_R - \Pi_R^{DAR}), \tag{N_{AB}}$$

where Π_A and Π_R are described in (15) and in (17), and

$$\Pi_A^{DAR} = s_2 r_u \pi_A(p_A) + s_2(1-s_2) r_u \left[\left(t_A - c_A^T\right) q(p_B) - \left(t_B - c_A^T\right) q(p_A)\right], \text{ and}$$

$$\begin{aligned}\Pi_R^{DAR} &= (r_R + (1-s_2)(1-r_R))\pi_R(p_R) + \\ &\quad (1-s_2)\left[(1-r_u)\left(t_{RB} - c_R^T\right) q(p_B) - (1-r_R)\left(t_{BR} - c_R^T\right)\gamma q(p_R)\right],\end{aligned}$$

where s_2 is given by (13). The first order conditions reduce to

$$\Pi_A - \Pi_A^{DAR} - \Pi_R + \Pi_R^{DAR} = 0. \qquad (N_{AR})$$

Similarly we can describe negotiations between network B and network R.

- **Sufficient Conditions**

Substituting (p$_i$), $i = A, B, R$ into (N_{AB}), (N_{AR}) and (N_{BR}), form a system of 3 equations in 6 unknowns, $(t_A, t_B, t_{AR}, t_{RA}, t_{BR}, t_{RB})$. For fixed t_{RA}, t_{RB} and t_B it is possible to solve for t_{AR}, t_{BR} and t_A as a function of t_{RA}, t_{RB} and t_B. The solution is unique and stable provided that the following matrix of partial derivatives is negative definite

$$Q_{(t_{RA}, t_{RB}, t_B)} = \begin{pmatrix} \frac{\partial N_{AR}}{\partial t_{AR}} & \frac{\partial N_{AR}}{\partial t_{BR}} & \frac{\partial N_{AR}}{\partial t_A} \\ \frac{\partial N_{BR}}{\partial t_{AR}} & \frac{\partial N_{BR}}{\partial t_{BR}} & \frac{\partial N_{BR}}{\partial t_A} \\ \frac{\partial N_{AB}}{\partial t_{AR}} & \frac{\partial N_{AB}}{\partial t_{BR}} & \frac{\partial N_{AB}}{\partial t_A} \end{pmatrix}.$$

We solve for a symmetric equilibrium of the simultaneous move game. At a symmetric equilibrium both sides of (23) are zero, so t_A and t_B become indeterminate. We assume that $t_A = t_B = c^T$ and $t_{RA} = t_{RA} = c_R^T$. In other words, we look at a scenario where the urban networks set a reciprocal access fee equal to the cost of terminating a phone call and the access fee that the rural network charges to a urban network is equal to each termination cost. We determine the prices that each network charges, as well as the access fee that the rural network pays to the urban network.

Example 1 *Suppose that*

$$q(p) = 10 - p$$

$$c^o = c^T = 1, \ c^F = 0$$

$$c_R^o = c_R^T = 2, \ \gamma = 0.5, \ \alpha = 50$$

The results are qualitatively the same with the ones obtained in the sequential move game.

Table 4: Simultaneous-Move Game: Symmetric Equilibrium ($c_R^F = 9$, $\pi_R = -4.5$)

	$r_U = 0.2$	$r_U = 0.2$	$r_U = 0.6$	$r_U = 0.9$
	$r_R = 0.7$	$r_R = 0.5$	$r_R = 0.5$	$r_R = 0.5$
$p_A = p_B$	6.1266	6.1406	5.7440	5.4186
p_R	5.5429	5.4065	5.8932	6.3592
$t_A = t_B$	1	1	1	1
$t_{RA} = t_{RB}$	2	2	2	2
$t_{AR} = t_{BR}$	-7.7137	-4.3739	-2.4274	-0.5634
$\Pi_A = \Pi_B$	3.5298	3.3608	5.3566	6.8904
Π_R	0.9327	-0.4032	-0.5669	-2.3721
Π_R^{DUR}	-1.1906	-0.3557	-1.4895	-2.3626
Π_U^{DUR}	1.4065	1.4076	4.4340	6.9000

Table 5: Simultaneous-Move Game: Symmetric Equilibrium ($c_R^F = 5$, $\pi_R = -0.5$)

	$r_U = 0.4$	$r_U = 0.2$
	$r_R = 0.7$	$r_R = 0.5$
	$c_R^F = 5$	$c_R^F = 5$
$p_A = p_B$	5.9083	5.6983
p_R	5.8864	6.1020
$t_A = t_B$	1	1
$t_{RA} = t_{RB}$	2	2
$t_{AR} = t_{BR}$	-5.4241	-1.5922
$\Pi_A = \Pi_B$	4.7863	5.8311
Π_R	3.4609	2.5974
$\Pi_R^{DUR},\ U = A, B$	1.5516	1.1857
$\Pi_U^{DUR},\ U = A, B$	2.8770	4.4194

4 Concluding Remarks

In this paper we investigate whether provision of universal service is sustainable without outside subsidies. We have extended the standard model of two competing interconnected networks, by including a third network that operates in a separate, high cost market, and have shown that in many instances interconnection will ensure universal service without government intervention or expensive subsidies. This is a simple, but an important insight, considering that the US government estimates that the amount of subsidies necessary to support universal service amounts to $5 billion per year.[13] Our results demonstrate that if urban customers place a large enough fraction of their total demand for calls to rural customers, then the rural network generates revenue from selling access that is high enough to cover its losses from providing telecommunication services. If this is the case in some markets, then there is no need to arrange for outside subsidies, since the market is viable on its own. Depending on the market power in the urban market there are different mechanisms in place that work in favor of cross-subsidization. Higher market power means higher profits that could be split across markets, but also means higher bargaining power vis-a-vis the rural network, whereas low market power implies very low profits in the urban market but at the same time, very weak position in the negotiations. A weakness of the current analysis is that our results are completely silent to whether prices will be comparable across markets. In the future we plan to investigate the effect of cross-market price constraints, similar to the ones in [1], on the negotiated access fees and on the equilibrium profits.

5 Appendix

Proof of Proposition 1.

The first order conditions of the above maximization problem reduce to

$$\Pi_A - \Pi_A^D = \Pi_R - \Pi_R^D. \tag{24}$$

$$\begin{aligned}
\Pi_A - \Pi_A^D &= (v - c_A^T) + (1 - r_R)t_{AR}\gamma - (1 - r_U)t_{RA} - [\gamma(1 - r_R) - (1 - r_U)]c_A^T - r_U(v - c_A^T) \\
&= (1 - r_U)(v - c_A^T) + (1 - r_R)t_{AR}\gamma - (1 - r_U)t_{RA} - [\gamma(1 - r_R) - (1 - r_U)]c_A^T \\
&= (1 - r_U)(v - c_A^T) - R_{AR} - [\gamma(1 - r_R) - (1 - r_U)]c_A^T
\end{aligned} \tag{25}$$

[13] *What Price Universal Service?*: Impact of Deleveraging Nationwide Urban/Rural Rates, Telecommunications Industries Analysis Project, Cambridge, MA., 1993.

$$
\begin{aligned}
\Pi_R - \Pi_R^D &= \gamma(v - c_R^T) - c_R^F - (1-r_R)t_{AR}\gamma + (1-r_U)t_{RA} + [\gamma(1-r_R) - (1-r_U)]c_R^T - \gamma r_R(v - c_R^T) - c_R^F \\
&= (1-r_R)\,\gamma(v - c_R^T) - c_R^F - (1-r_R)t_{AR}\gamma + (1-r_U)t_{RA} + [\gamma(1-r_R) - (1-r_U)]c_R^T \\
&= (1-r_R)\,\gamma(v - c_R^T) + R_{AR} + [\gamma(1-r_R) - (1-r_U)]c_R^T \qquad (26)
\end{aligned}
$$

Combining (24), (25), and (26) we obtain

$$
\begin{aligned}
(1-r_U)(v - c_A^T) - R_{AR} - [\gamma(1-r_R) - (1-r_U)]c_A^T &= (1-r_R)\,\gamma(v - c_R^T) + R_{AR} + [\gamma(1-r_R) - (1-r_U)]c_R^T \\
2R_{AR} &= (1-r_U)(v - c_A^T) - (1-r_R)\,\gamma(v - c_R^T) \\
&\quad -[\gamma(1-r_R) - (1-r_U)]c_A^T - [\gamma(1-r_R) - (1-r_U)]c_R^T
\end{aligned}
$$

$$
\begin{aligned}
2R_{AR} &= [(1-r_U) - (1-r_R)\,\gamma]\,v - c_A^T\,[(1-r_U) + \gamma(1-r_R) - (1-r_U)] \\
&\quad -c_R^T\,[-(1-r_R)\,\gamma + \gamma(1-r_R) - (1-r_U)] \\
&= [(1-r_U) - (1-r_R)\,\gamma]\,v - c_A^T\gamma(1-r_R) + c_R^T(1-r_U)
\end{aligned}
$$

or

$$
R_{AR} = 0.5\left[((1-r_U) - (1-r_R)\,\gamma)\,v - c_A^T\gamma(1-r_R) + c_R^T(1-r_U)\right].
$$

Proof of Proposition 3.

Recall that

$$
\Pi_R = \gamma(v - c_R^T) - c_R^F + R_{AR} + [\gamma(1-r_R) - (1-r_U)]c_R^T,
$$

then by substituting in R_{AR} we get

$$
\begin{aligned}
\Pi_R &= \gamma(v - c_R^T) - c_R^F + 0.5\left[((1-r_U) - (1-r_R)\,\gamma)\,v - c_A^T\gamma(1-r_R) + c_R^T(1-r_U)\right] \\
&\quad +[\gamma(1-r_R) - (1-r_U)]c_R^T - c_R^F \\
&= (0.5\,((1-r_U) - (1-r_R)\,\gamma) + \gamma)\,v - 0.5\gamma(1-r_R)c_A^T \\
&\quad +[\gamma(1-r_R) - \gamma - (1-r_U) + 0.5(1-r_U)]c_R^T - c_R^F
\end{aligned}
$$

$$
\begin{aligned}
\Pi_R &= (0.5(1-r_U) - 0.5\,(1-r_R)\,\gamma + \gamma)\,v - 0.5\gamma(1-r_R)c_A^T \\
&\quad +[\gamma(1-r_R) - \gamma - (1-r_U) + 0.5(1-r_U)]c_R^T - c_R^F \\
&= (0.5\,((1-r_U) + 0.5r_R\gamma + 0.5\gamma))\,v - 0.5\gamma(1-r_R)c_A^T - [r_R\gamma + 0.5(1-r_U)]c_R^T - c_R^F \\
&= (0.5\,((1-r_U) + 0.5(1+r_R)\gamma))\,v - 0.5\gamma(1-r_R)c_A^T - [r_R\gamma + 0.5(1-r_U)]c_R^T - c_R^F
\end{aligned}
$$

Similarly by substituting in R_{AR} in Π_A we get

$$
\begin{aligned}
\Pi_A &= (v - c_A^T) - 0.5\left[((1-r_U) - (1-r_R)\,\gamma)\,v - c_A^T\gamma(1-r_R) + c_R^T(1-r_U)\right] \\
&\quad -[\gamma(1-r_R) - (1-r_U)]c_A^T \\
&= [1 - 0.5\,((1-r_U) - (1-r_R)\,\gamma)]\,v - c_A^T\,[\gamma(1-r_R) - (1-r_U) + 1 - 0.5\gamma(1-r_R)] \\
&\quad -0.5c_R^T(1-r_U) \\
&= [0.5(1+r_U) + (1-r_R)\,\gamma]\,v - c_A^T\,[0.5\gamma(1-r_R) + r_U] - 0.5c_R^T(1-r_U)
\end{aligned}
$$

Summary:

$$
\begin{aligned}
\Pi_A &= [0.5(1+r_U) + (1-r_R)\,\gamma]\,v - c_A^T\,[0.5\gamma(1-r_R) + r_U] - 0.5c_R^T(1-r_U) \\
\Pi_R &= (0.5\,((1-r_U) + 0.5(1+r_R)\gamma))\,v - 0.5\gamma(1-r_R)c_A^T - [r_R\gamma + 0.5(1-r_U)]c_R^T - c_R^F,
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
\Pi_R &= 0.5\gamma(1+r_R)(v - c_R^T) + 0.5(1-r_U)(v - c_A^T) - c_R^F \\
&\quad -0.5\,[(1-r_U) - \gamma(1-r_R)]\,(c_R^T - c_A^T)
\end{aligned}
$$

and

$$
\begin{aligned}
\Pi_A &= 0.5(1+r_U)(v - c_A^T) + 0.5\gamma(1-r_R)(v - c_R^T) \\
&\quad -0.5\,[(1-r_U) - \gamma(1-r_R)]\,(c_R^T - c_A^T).
\end{aligned}
$$

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